

Example: Regular Pumping Lemma_{JP}

The following is a walk-through of the JFLAP Regular Pumping Lemma Game for the lemma

$$L = \{ww^R : w \in (a, b)^*\}$$

Recall that if L is a regular language then there exists an integer $m > 0$ such that any $w \in L$ with $|w| \geq m$ can be decomposed as the concatenation $w = xyz$, with $|xy| \leq m$, $|y| \geq 1$, and $xy^iz \in L$ for all $i \geq 0$.

Consider what characteristics of a string are necessary for successful decomposition into three concatenated components, the middle of which can be eliminated or repeated.

Walkthrough (see RPL_wwr.jff)

1. Open JFLAP and select *Regular Pumping Lemma*



2. Select “Computer goes first” and choose *Select* to the right of “ $L = \{ww^R : w \in (a, b)^*\}$ ”

JFLAP : (RPL_wvr.jff)

File Help

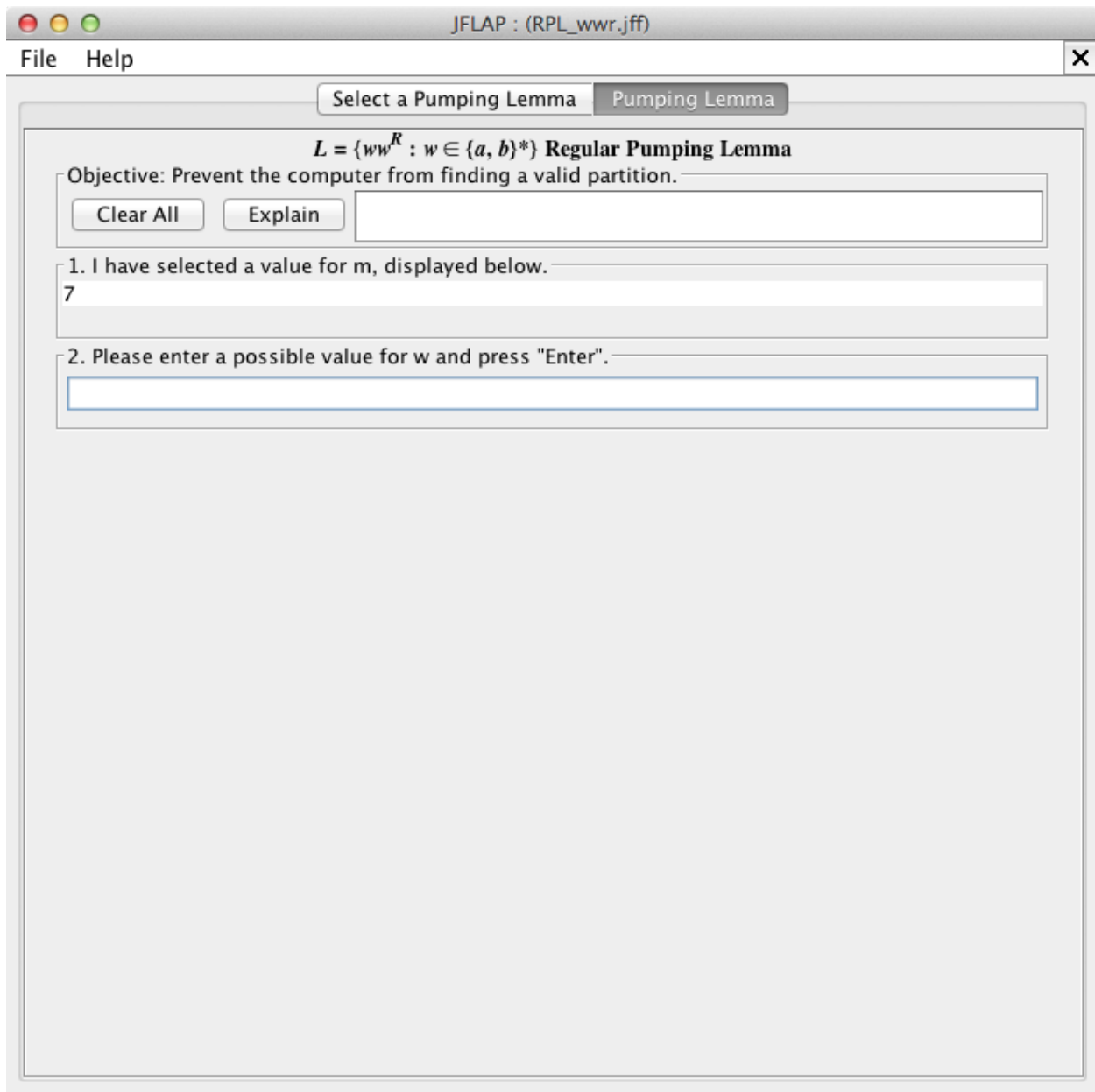
Select a Pumping Lemma

First choose who makes the first move.

You go first Computer goes first

Then select a lemma.

$L = \{a^n b^n : n \geq 0\}$	Select
$L = \{w \in \{a, b\}^* : n_a(w) < n_b(w)\}$	Select
$L = \{ww^R : w \in \{a, b\}^*\}$	Select
$L = \{(ab)^n a^k : n > k, k \geq 0\}$	Select
$L = \{a^n b^k c^{n+k} : n \geq 0, k \geq 0\}$	Select
$L = \{a^n b^l a^k : n > 5, l > 3, k \leq l\}$	Select
$L = \{a^n : n \text{ is even}\}$	Select
$L = \{a^n b^k : n \text{ is odd or } k \text{ is even.}\}$	Select
$L = \{bba(ba)^n a^{n-1}\}$	Select
$L = \{b^5 w : w \in \{a, b\}^*, 2n_a(w) = 3n_b(w)\}$	Select
$L = \{b^5 w : w \in \{a, b\}^*, (2n_a(w) + 5n_b(w)) \bmod 3 = 0\}$	Select
$L = \{b^k (ab)^n (ba)^n : k \geq 4, n = 1, 2, \dots\}$	Select
$L = \{(ab)^{2n} : n = 1, 2, \dots\}$	Select



Notice that the computer has selected the value 7 for m.

3. Consider strings of length 7 or greater in language L.

The string ww^R is the concatenation of two strings, w and w^R , and that $|w| = |w^R|$. Therefore all strings in L must have even length.

Enter a candidate value for w of length 8, such as aabaabaa.

Select a Pumping Lemma Pumping Lemma

$L = \{ww^R : w \in \{a, b\}^*\}$ Regular Pumping Lemma

Objective: Prevent the computer from finding a valid partition.

Clear All

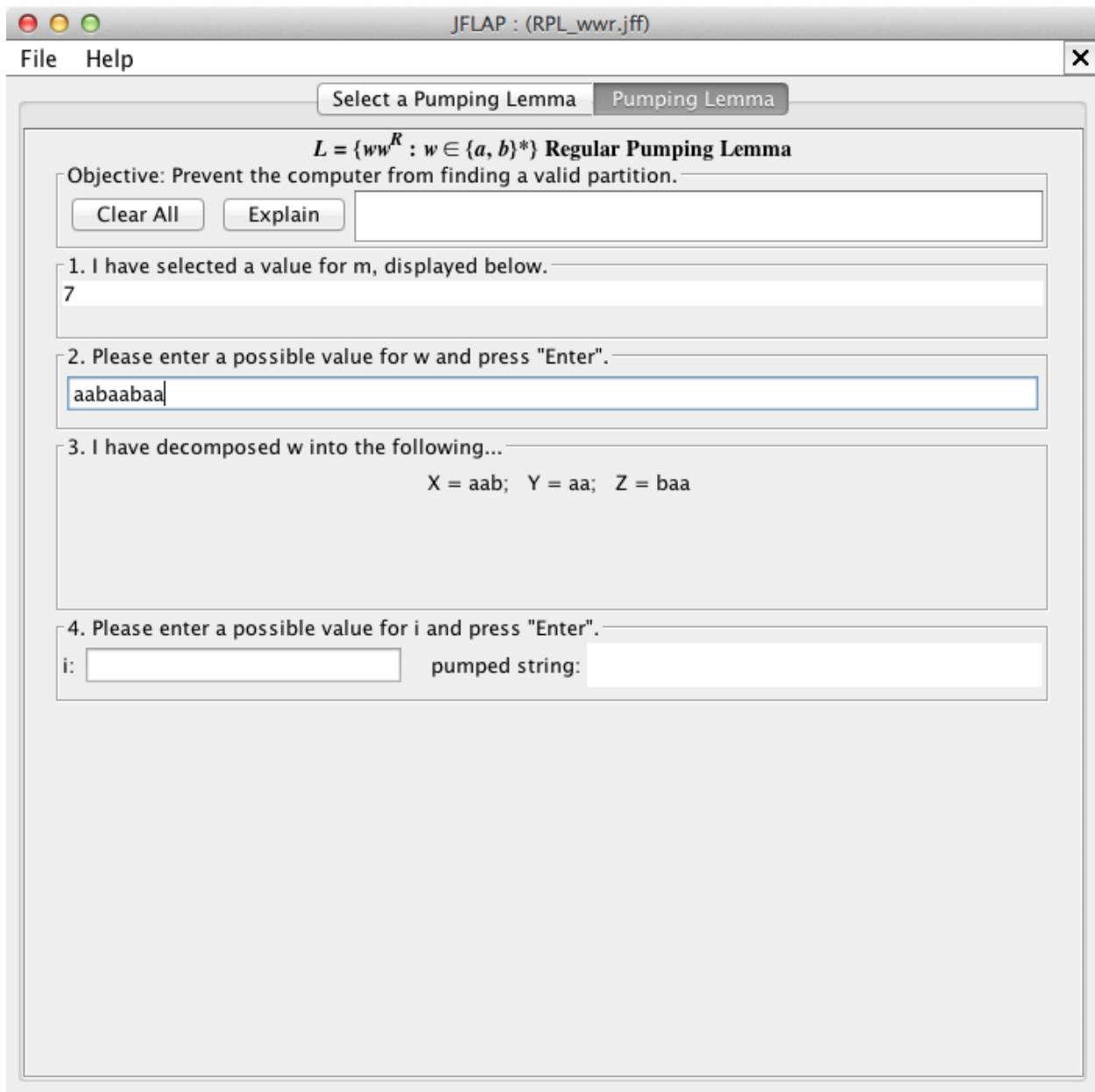
Explain

1. I have selected a value for m, displayed below.

7

2. Please enter a possible value for w and press "Enter".

aabaabaa



The decomposition chosen by the computer appears to be one that fits all of the requirements of the pumping lemma.

4. Enter any value for i , such as 0, to see the pumped string.

The screenshot shows the JFLAP software window titled "JFLAP : (RPL_wwr.jff)". The window has a menu bar with "File" and "Help". Below the menu bar are two tabs: "Select a Pumping Lemma" and "Pumping Lemma". The main content area is titled " $L = \{ww^R : w \in \{a, b\}^*\}$ Regular Pumping Lemma". Below the title is the objective: "Objective: Prevent the computer from finding a valid partition." There are two buttons, "Clear All" and "Explain", and a text input field. The first step is "1. I have selected a value for m , displayed below." with the value "7" entered. The second step is "2. Please enter a possible value for w and press 'Enter'." with the value "aabaabaa" entered. The third step is "3. I have decomposed w into the following..." with the decomposition "X = aab; Y = aa; Z = baa" displayed. The fourth step is "4. Please enter a possible value for i and press 'Enter'." with the value "0" entered in the "i:" field and an empty "pumped string:" field.

JFLAP : (RPL_wwr.jff)

File Help

Select a Pumping Lemma Pumping Lemma

$L = \{ww^R : w \in \{a, b\}^*\}$ Regular Pumping Lemma

Objective: Prevent the computer from finding a valid partition.

Clear All Explain

1. I have selected a value for m , displayed below.

7

2. Please enter a possible value for w and press "Enter".

aabaabaa

3. I have decomposed w into the following...

X = aab; Y = aa; Z = baa

4. Please enter a possible value for i and press "Enter".

i: 0 pumped string:

JFLAP : (RPL_wvr.jff)

File Help

Select a Pumping Lemma Pumping Lemma

$L = \{ww^R : w \in \{a, b\}^*\}$ Regular Pumping Lemma

Objective: Prevent the computer from finding a valid partition.

Clear All Explain My Attempts
 1: X = aab; Y = aa; Z = baa; I = 0; Failed

1. I have selected a value for m, displayed below.
 7

2. Please enter a possible value for w and press "Enter".
 aabaabaa

3. I have decomposed w into the following...
 X = aab; Y = aa; Z = baa

4. Please enter a possible value for i and press "Enter".
 i: 0 pumped string: aabbbaa

5. Animation

$$w = \begin{matrix} x & y & z \\ aab & aa & baa \end{matrix}$$

$xy^0z = a^2b^2a^2 = aabbbaa$ is in the language. Please try again.

Step Restart

5. Step through the animation to see how the pumped string is constructed.

JFLAP : (RPL_wvr.jff)

File Help

Select a Pumping Lemma Pumping Lemma

$L = \{ww^R : w \in \{a, b\}^*\}$ Regular Pumping Lemma

Objective: Prevent the computer from finding a valid partition.

Clear All Explain My Attempts:
1: X = aab; Y = aa; Z = baa; I = 0; Failed

1. I have selected a value for m, displayed below.
7

2. Please enter a possible value for w and press "Enter".
aabaabaa

3. I have decomposed w into the following...
X = aab; Y = aa; Z = baa

4. Please enter a possible value for i and press "Enter".
i: 0 pumped string: aabbaa

5. Animation

x y z
w = aab aa baa
aab

$xy^0z = a^2b^2a^2 = aabbaa$ is in the language. Please try again.

Step Restart

Select a Pumping Lemma Pumping Lemma

$L = \{ww^R : w \in \{a, b\}^*\}$ Regular Pumping Lemma

Objective: Prevent the computer from finding a valid partition.

My Attempts:
1: X = aab; Y = aa; Z = baa; I = 0; *Failed*

1. I have selected a value for m, displayed below.

2. Please enter a possible value for w and press "Enter".

3. I have decomposed w into the following...
X = aab; Y = aa; Z = baa

4. Please enter a possible value for i and press "Enter".
i: pumped string:

5. Animation

x y z
w = aab aa baa

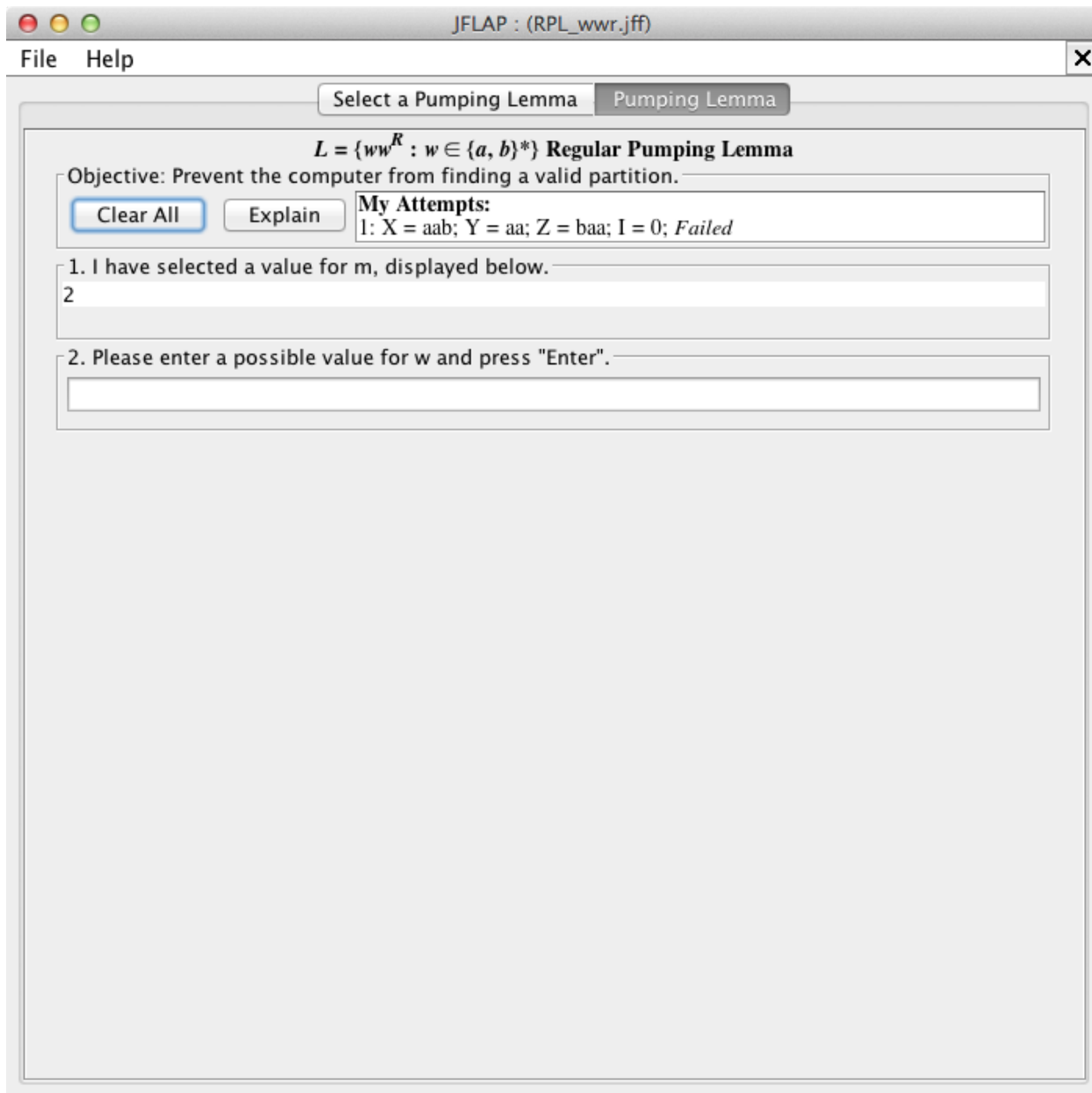
aabbbaa

$xy^0z = a^2b^2a^2 = aabbbaa$ is in the language. Please try again.

6. Now choose *Clear All* to try another string.

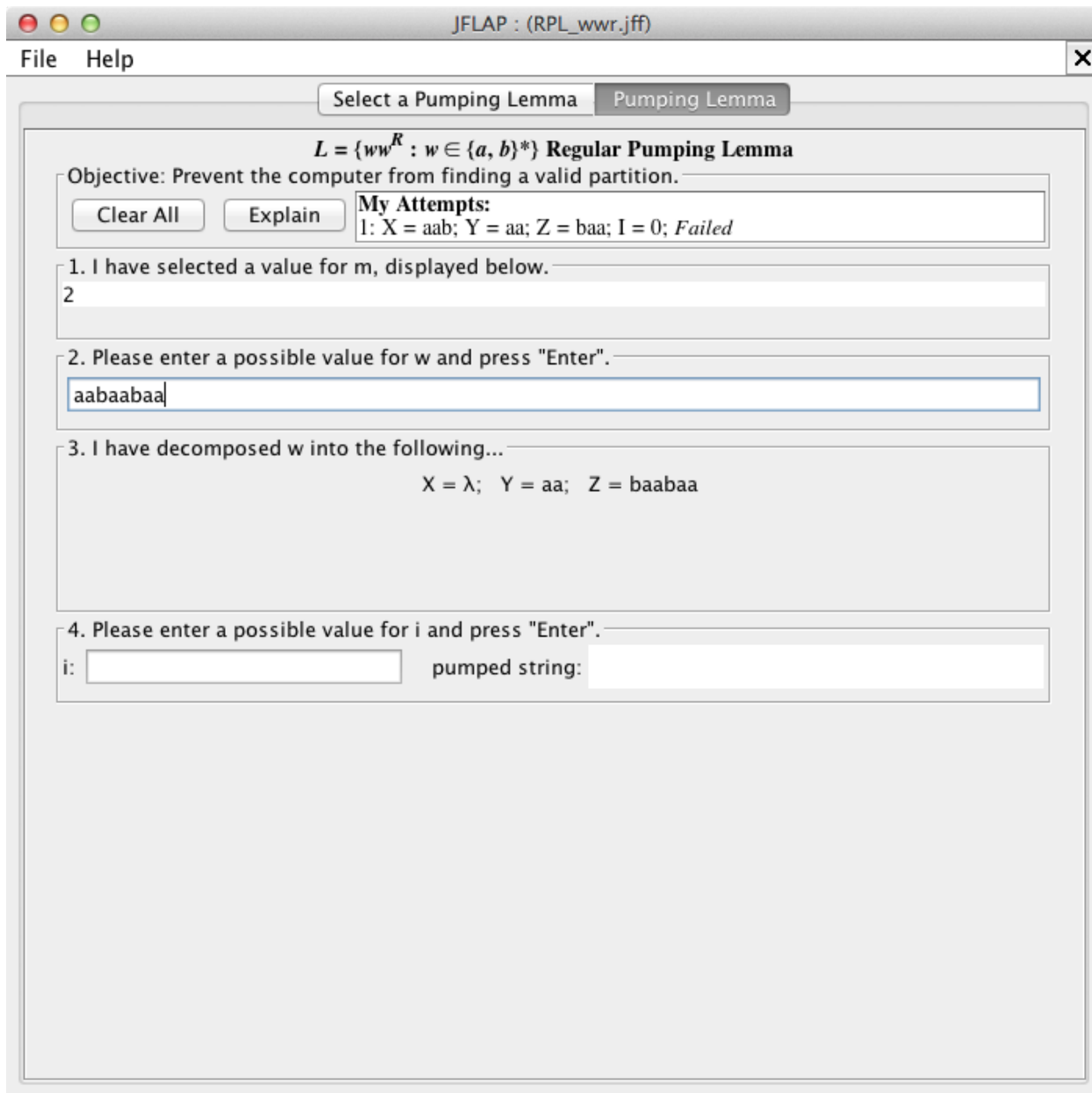
The screenshot shows the JFLAP software window titled "JFLAP : (RPL_wvr.jff)". The interface includes a menu bar with "File" and "Help", and a toolbar with "Select a Pumping Lemma" and "Pumping Lemma" buttons. The main content area is titled "Regular Pumping Lemma" and contains the following elements:

- Objective:** Prevent the computer from finding a valid partition.
- Clear All** and **Explain** buttons.
- My Attempts:** A text box containing "1: X = aab; Y = aa; Z = baa; I = 0; Failed".
- Step 1:** "I have selected a value for m, displayed below." with a text box containing "7".
- Step 2:** "Please enter a possible value for w and press 'Enter'." with a text box containing "aabaabaa".
- Step 3:** "I have decomposed w into the following..." with the text "X = aab; Y = aa; Z = baa".
- Step 4:** "Please enter a possible value for i and press 'Enter'." with a text box for "i:" containing "0" and a text box for "pumped string:" containing "aabbbaa".
- Step 5: Animation**
$$\begin{array}{ccc} \mathbf{x} & \mathbf{y} & \mathbf{z} \\ \mathbf{w} = & \mathbf{aab} & \mathbf{aa} & \mathbf{baa} \\ & & & \\ & & & \mathbf{aabbbaa} \end{array}$$
- At the bottom, the text " $xy^0z = a^2b^2a^2 = aabbbaa$ is in the language. Please try again." is displayed, along with **Step** and **Restart** buttons.



This time the computer has selected $m = 2$.

7. Consider the pumping lemma requirement that $|xy| \leq m$. Consider whether the previously constructed string, aabaabaa, would produce strings in language L when pumped. Try this string again.



8. Consider which values for i would result in a string that is not in language L .

All values of $i \neq 1$ will work. That is, only in the case where $i = 1$ (the original string) is $xy^iz \in L$.

For this example, enter 0.

JFLAP : (RPL_wvr.jff)

File Help

Select a Pumping Lemma Pumping Lemma

$L = \{ww^R : w \in \{a, b\}^*\}$ Regular Pumping Lemma

Objective: Prevent the computer from finding a valid partition.

Clear All Explain My Attempts: 2: X = λ ; Y = aa; Z = baabaa; I = 0; Won

1. I have selected a value for m, displayed below.

2

2. Please enter a possible value for w and press "Enter".

aabaabaa

3. I have decomposed w into the following...

X = λ ; Y = aa; Z = baabaa

4. Please enter a possible value for i and press "Enter".

i: 0 pumped string: baabaa

5. Animation

$w = \overset{x}{_} \overset{y}{aa} \overset{z}{baabaa}$

$xy^0z = ba^2ba^2 = baabaa$ is NOT in the language. YOU WIN!

Step Restart

9. Select *Step* to see how the pumped string was constructed.

JFLAP : (RPL_wvr.jff)

File Help

Select a Pumping Lemma Pumping Lemma

$L = \{ww^R : w \in \{a, b\}^*\}$ Regular Pumping Lemma

Objective: Prevent the computer from finding a valid partition.

Clear All Explain My Attempts:
2: X = λ ; Y = aa; Z = baabaa; I = 0; Won

1. I have selected a value for m, displayed below.
2

2. Please enter a possible value for w and press "Enter".
aabaabaa

3. I have decomposed w into the following...
X = λ ; Y = aa; Z = baabaa

4. Please enter a possible value for i and press "Enter".
i: 0 pumped string: baabaa

5. Animation

x y z
w = _ aa baabaa

$xy^0z = ba^2ba^2 = baabaa$ is NOT in the language. YOU WIN!

Step Restart

JFLAP : (RPL_wvr.jff)

File Help

Select a Pumping Lemma Pumping Lemma

$L = \{ww^R : w \in \{a, b\}^*\}$ Regular Pumping Lemma

Objective: Prevent the computer from finding a valid partition.

Clear All Explain My Attempts: 2: X = λ ; Y = aa; Z = baabaa; I = 0; Won

1. I have selected a value for m, displayed below.

2

2. Please enter a possible value for w and press "Enter".

aabaabaa

3. I have decomposed w into the following...

X = λ ; Y = aa; Z = baabaa

4. Please enter a possible value for i and press "Enter".

i: 0 pumped string: baabaa

5. Animation

$$w = \overset{x}{_} \overset{y}{aa} \overset{z}{baabaa}$$

$$baabaa$$

$xy^0z = ba^2ba^2 = baabaa$ is NOT in the language. YOU WIN!

Step Restart

10. Select *Explain* for a general explanation.

$L = \{ww^R : w \in \{a, b\}^*\}$ **Regular Pumping Lemma**

Objective: Prevent the computer from finding a valid partition.

Unfortunately no valid partition of w exists.
 For any m value, a possible value for w is " $a^m bba^m$ ". The y value thus

1. I have selected a value for m , displayed below.

2. Please enter a possible value for w and press "Enter".

3. I have decomposed w into the following...
 $X = \lambda; Y = aa; Z = baabaa$

4. Please enter a possible value for i and press "Enter".
 i: pumped string:

5. Animation

$$w = \overset{x}{_} \overset{y}{aa} \overset{z}{baabaa}$$

$$baabaa$$

$xy^0z = ba^2ba^2 = baabaa$ is NOT in the language. YOU WIN!

Note that you may need to scroll to read the full explanation, which is reproduced here.

Unfortunately no valid partition of w exists.

For any m value, a possible value for w is " $a^m bba^m$ ". The y value thus would be a multiple of "a" in ' w ' and not in ' w^R '. If $i = 0$, then the total string becomes at most " $a^{m-1} bba^m$ ", which is not in the language. Thus, the language is not regular.

My Attempts:

2: $X = \lambda; Y = aa; Z = baabaa; I = 0$; *Won*

1: $X = aab; Y = aa; Z = baa; I = 0$; *Failed*