

Converting a DFA to a Regular Expression

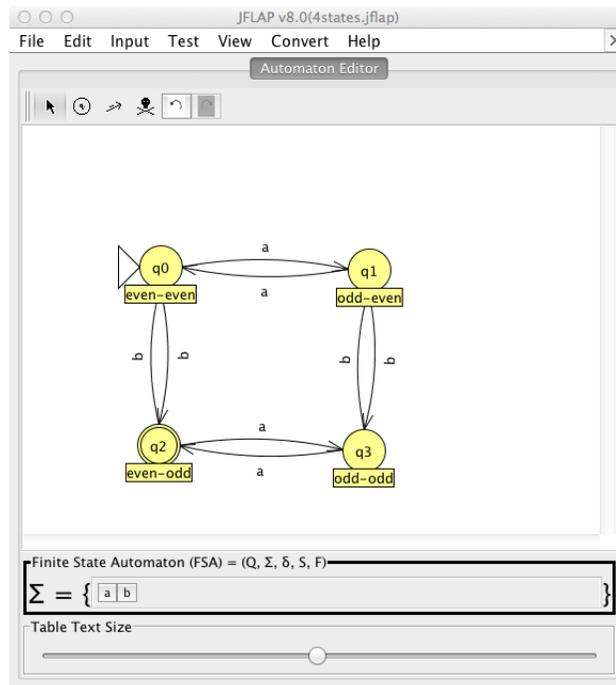
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1 DFA Example

We are given a DFA. In this Exercise, we will convert the DFA to a regular expression. Consider the language

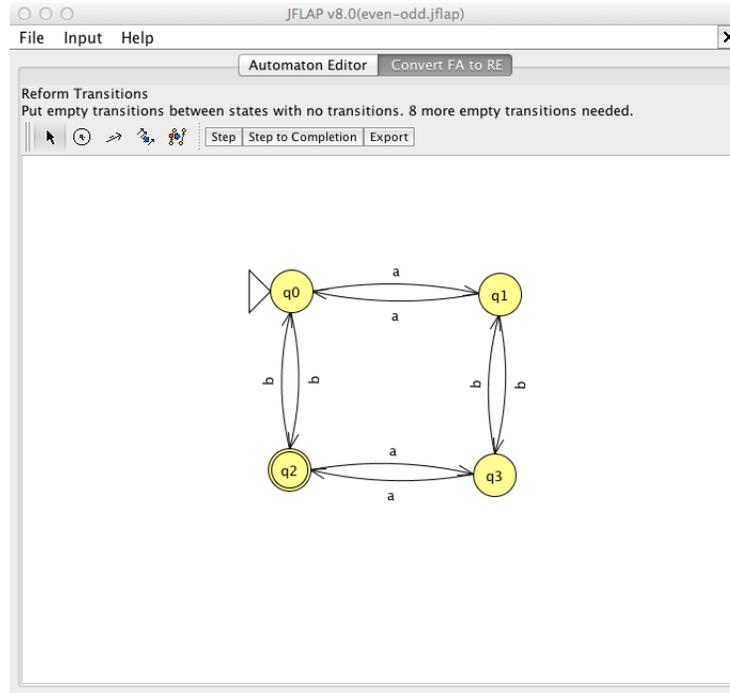
$$L = \{w \in \Sigma^* | w \text{ has an even number of } a\text{'s and an odd number of } b\text{'s}\}.$$

A DFA for L is as shown below.

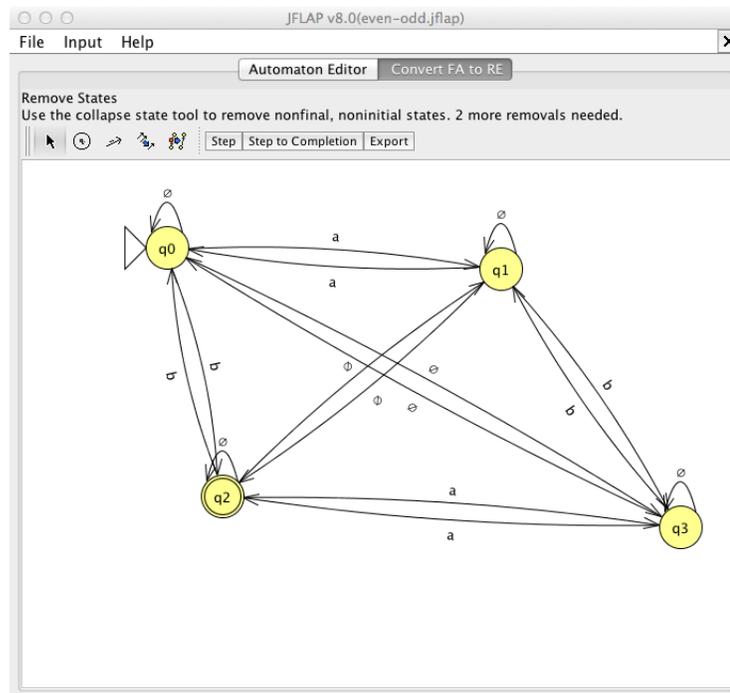


Below, you'll use JFLAP to build a regular expression for this DFA that represents the language L .

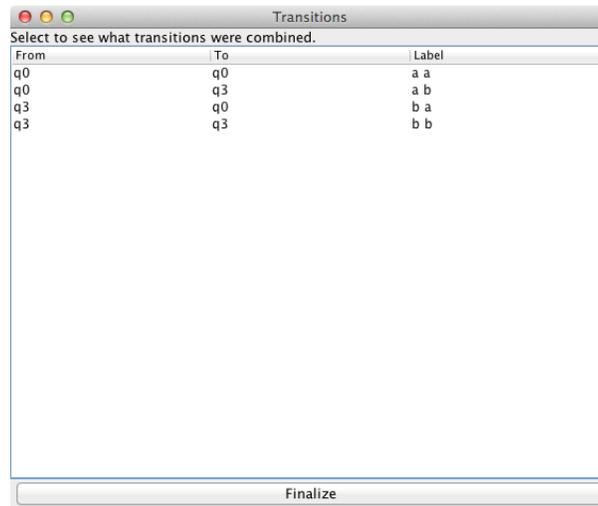
1. This conversion algorithm requires a generalized transition graph (GTG) to be constructed. In a GTG, there is a single final state, an initial state that is not final, and exactly one transition for each pair of states (including loops).
2. Load this DFA in JFLAP by opening the file even-odd.jflap. Choose Convert from the menu and Click on Convert to RE. You'll get a new window as below.



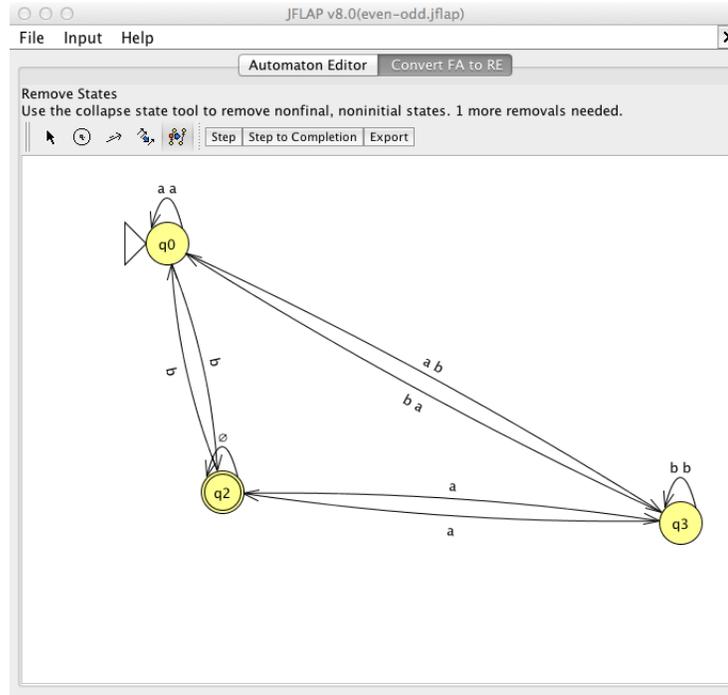
3. Review the message in the window. Since there is only one final state that is distinct from the initial state, it only remains to add transitions. JFLAP tells you there are 8 empty transitions needed. Four of these are loops. What are the other four?
4. You can click on the button **Step** to have JFLAP do this for you. However, you should practice adding the 8 transitions manually. You get a GTG as below.



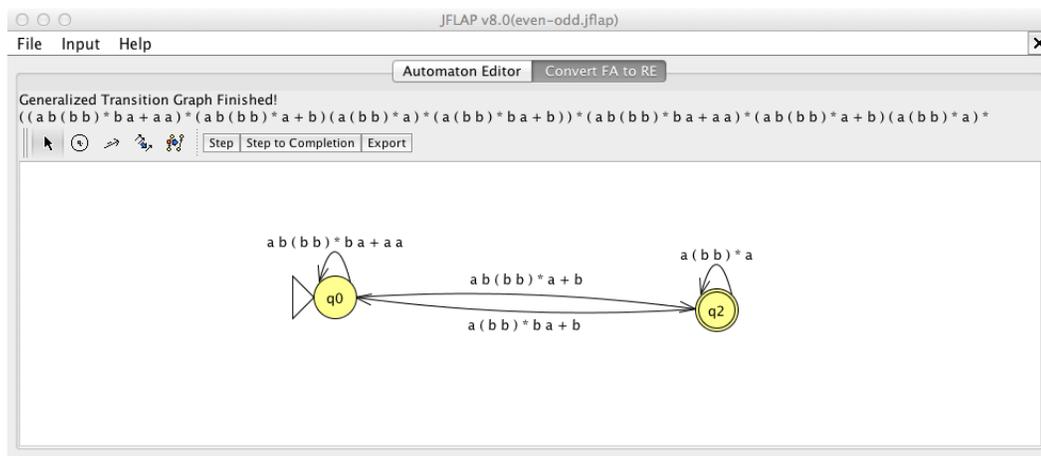
5. At this stage you already have exactly one transition between each pair of states (including loops).
6. The next step is to collapse states (one at a time) that are not the final and the initial states.
7. Click on the State Collapser tool. Then choose a state, $q1$. You'll get a window labeled Transitions as below.



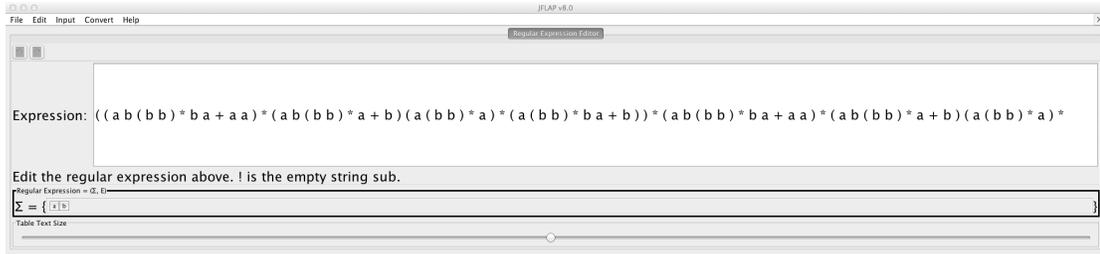
8. Review the contents of this window carefully. To collapse $q1$ we have to preserve the effect of all transitions that go through $q1$. E.g. there are transitions labeled a from $q0$ to $q1$ and b from $q1$ to $q3$. This is shown on the second row of contents in the window Transitions. This means that we should add an RE labeled ab from $q0$ to $q3$ when $q1$ is collapsed. Select the second row. This highlights the the relevant transitions in the GTG. Similarly, select each row of the Transitions window and study and understand the corresponding highlighted transitions in the GTG. When you are satisfied, click the Finalize button to collapse $q1$ and add new transitions. You should now have a window as below.



9. Now repeat the above process to remove another state. Try to remove q_0 and q_2 . What happens?
10. Next remove q_3 and inspect the Transitions window. Repeat steps as above. As you inspect each row, study the RE labels carefully.
11. Click on Finalize to get the following window. Inspect the RE in this carefully.



12. Click on Step to get the RE in a new window, as below. Explain why this is the RE for L .



2 Additional Exercises

1. Repeat the above steps for the language
 $L_1 = \{w \in \Sigma^* \mid w \text{ has an odd number of } a\text{'s and an odd number of } b\text{'s}\}$
2. Repeat the above steps for the language
 $L_2 = \{w \in \Sigma^* \mid \text{The number of } a\text{'s in } w \text{ is a multiple of } 3\}$

3 References

1. Introduction to the Theory of Computation (Third Edition), Michael Sipser. Cengage Learning. 2013.
2. JFLAP - An Interactive Formal Languages and Automata Package, Susan H. Rodger and Thomas W Finley. Jones and Bartlett Publishers. 2006