Minimization of DFA

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1 Introduction

We use JFLAP to apply the DFA minimization algorithm. Given a DFA M_1, we find a DFA M_2 with the smallest number of possible states such that M_1 and M_2 are equivalent.

The minimization algorithm works by identifying indistinguishable states and grouping them into single states. Informally, two states are *indistinguishable* if their behavior with transitions and acceptance is essentially the same. Formally, two states p and q are indistinguishable if for every word w, the executions of w from p and q are both accepting states or both non-accepting states. Two states are *distinguishable* if they are not indistinguishable. For example, an accept state is distinguishable from a non-accept state since λ takes one to accept and another to non-accept. Rather than configurations of strings, we can discover distinguishability with one symbol at a time. Thus if there are states p and q and a symbol a such that a takes p and q to distinguishable states, then p and q are themselves distinguishable. It turns out that the relation of indistinguishability is an equivalence relation. Thus the minimization of a DFA is achieved by grouping the equivalence classes of states into single states. We discuss two examples below.

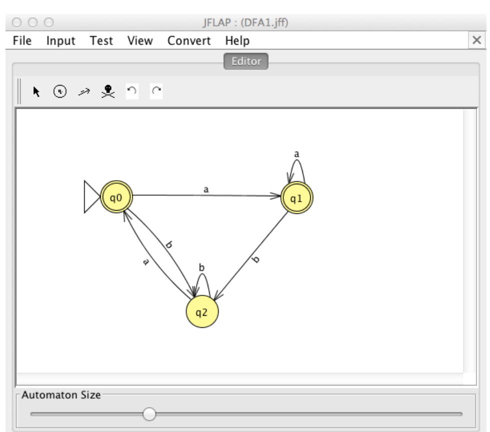
2 Example 1

Let us consider the language $L_1 = \{w \in \Sigma^* | w \text{ is } \lambda \text{ or } w \text{ ends in } a\}$.

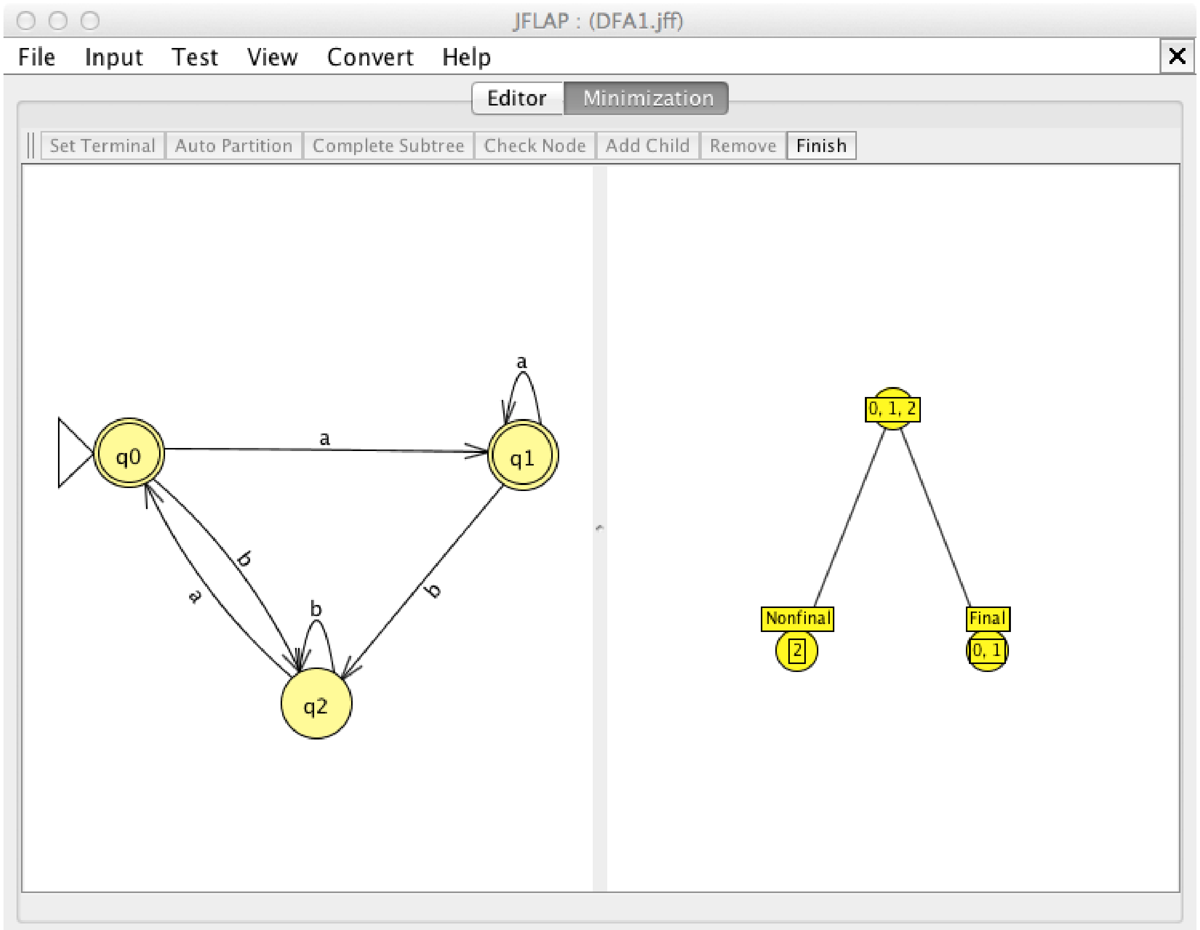
1. Create a three state DFA as below. From the initial state $q0$, which is also an accept state, an a takes in to $q1$ (an accept state) and a b takes it to $q2$. All the incoming transitions to $q0$ and $q1$ have labels a, while those to $q2$ have label b.

2. Verify that this DFA accepts L_1.

3. From what we said above, it appears that $q0$ and $q1$ have similar behavior. Are those two states indistinguishable?

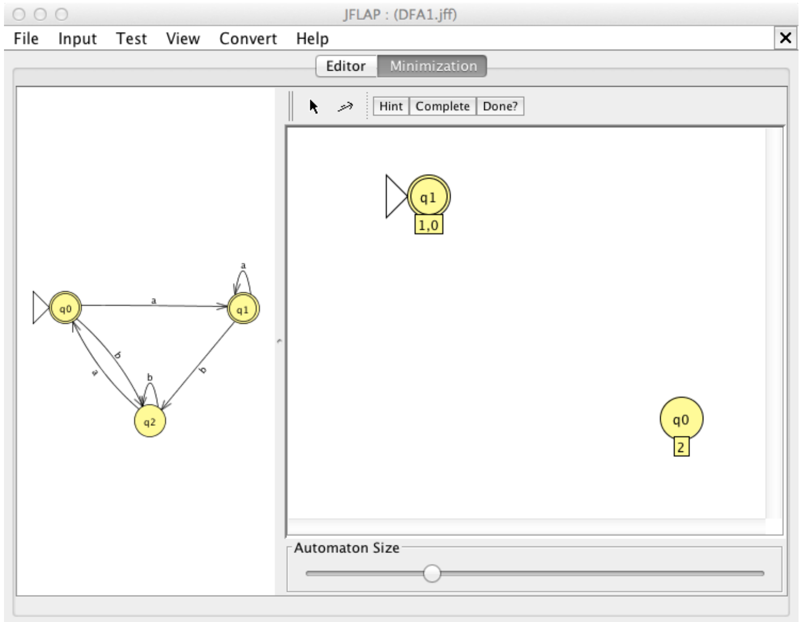


4. In JFLAP, choose Convert from the menu and click on minimize DFA. The window splits in two and you see the DFA on left and a tree diagram on right. The root of the tree has three states, and two branches show 2 in one node and 0,1 in the other. These are groups of possible indistinguishable states. See below.



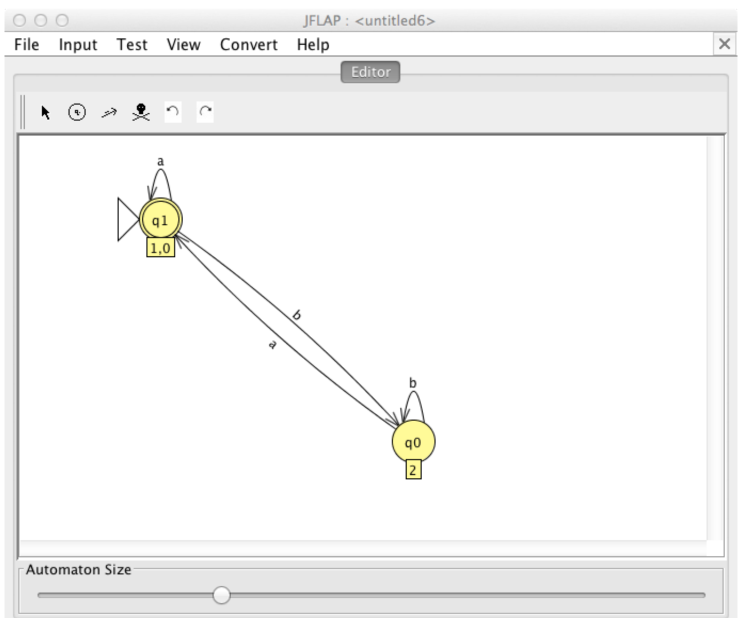
5. For this small example, JFLAP does not have further choice other than to click on the button $\boxed{\text{Finish}}$. JFLAP presents another window, as below, where the DFA on the right has two states, with old state labels 1,0 combined as a new state $q1$. Similarly for $q2$.

6. We still need to add transitions. You can click on $\boxed{\text{Hint}}$ to have JFLAP do it or add your own. Inspect the original DFA on the left to see how the labels help you to create the transitions.



7. You can also click on $\boxed{\text{Done?}}$ to see how many more transitions remain to be added.

8. Once you are satisfied, click on $\boxed{\text{Done?}}$ to see the answer, as below.

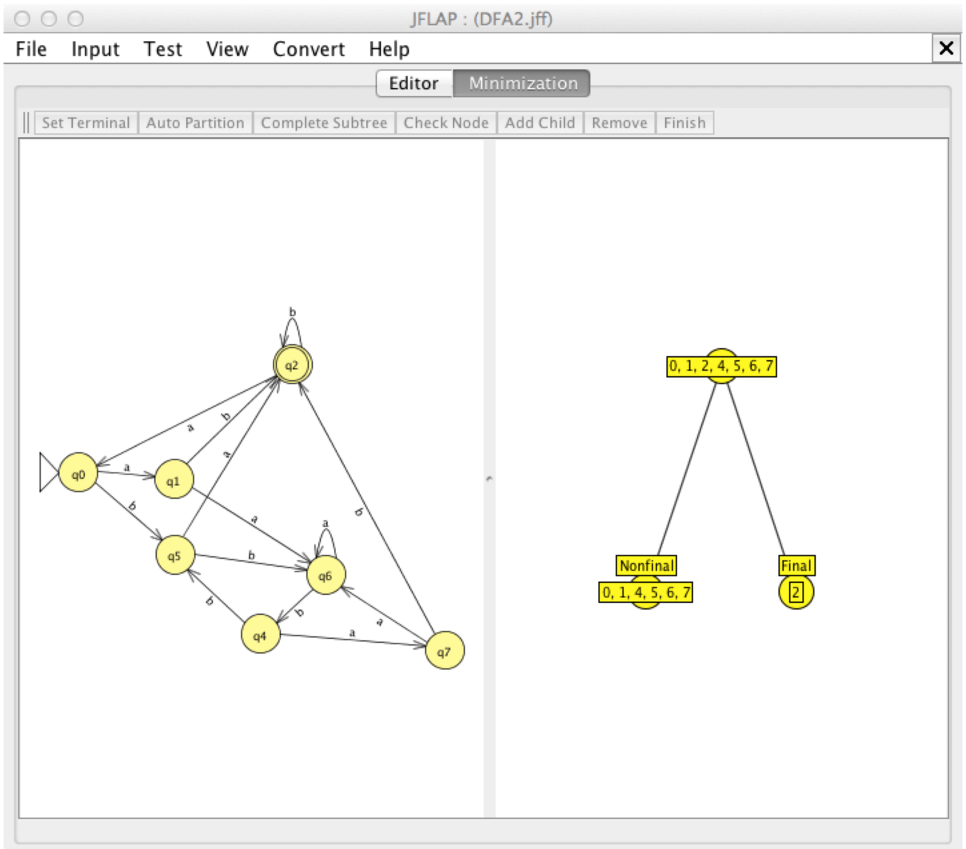


9. Verify that this 2-state DFA is equivalent to the 3-state DFA above that you started with.

3 Example 2

For our second example, consider the DFA shown below in the left window.

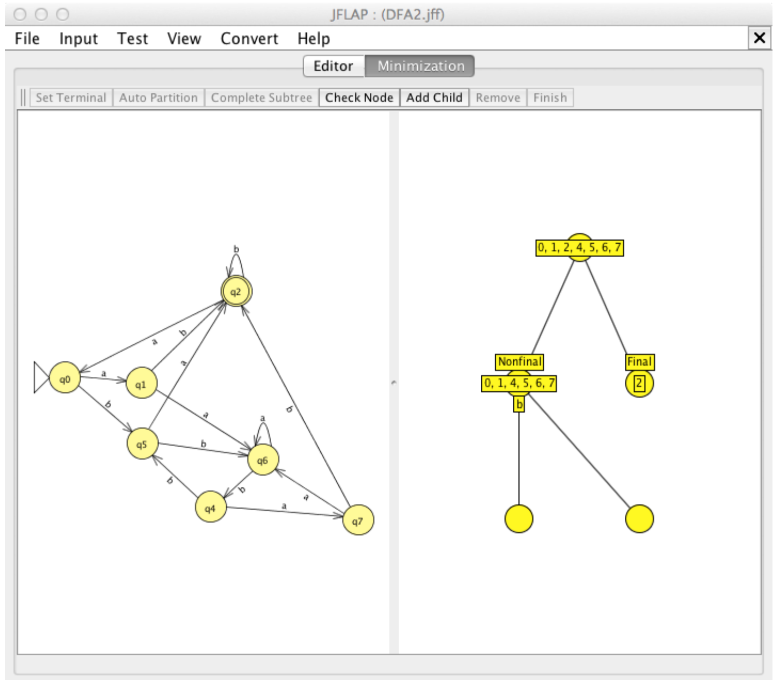
1. Load the file DFA2.jff. Choose Convert from the menu and Click on Minimize DFA.



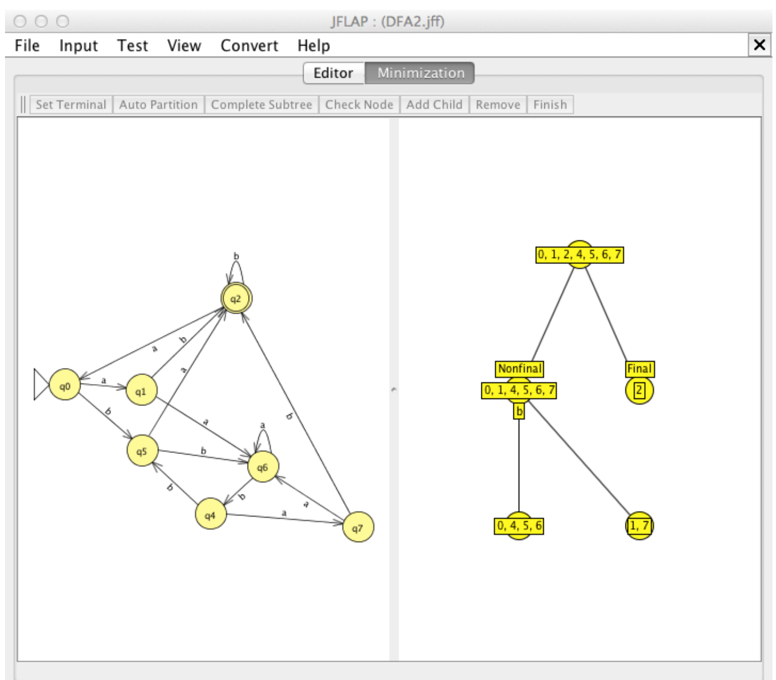
2. The tree diagram on the right shows the two groups of states: Nonfinal and Final. The final state is distinguishable from the others.

3. To find if there are more distinguishable states among $\{0, 1, 4, 5, 6, 7\}$ click on the left leaf in the tree. This highlights the corresponding states on the left. Click on Set Terminal. A dialog box asks for an input. Try a. It tells you that that group does not spilt the partition further. Try b.

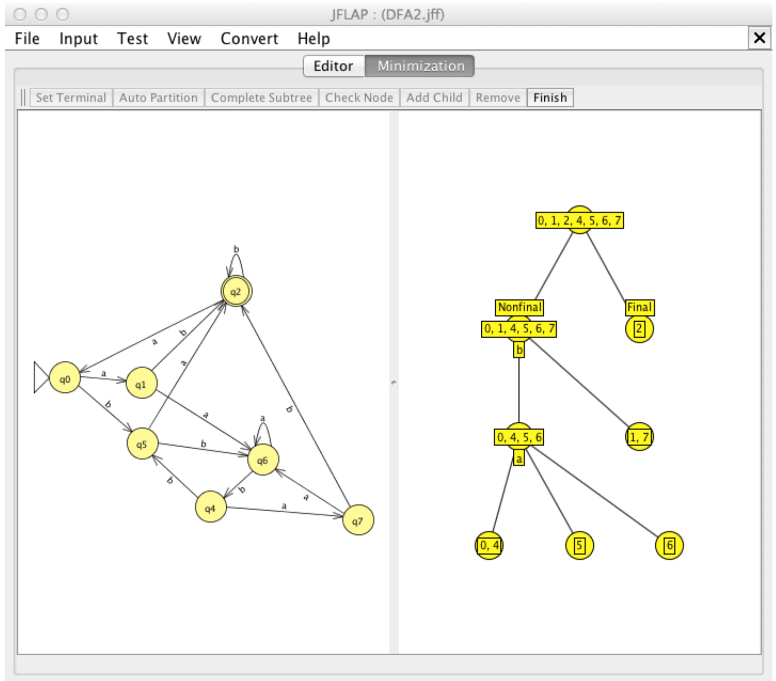
4. This presents two child nodes, as below. We now have to find distinguishable states in $\{0, 1, 4, 5, 6, 7\}$. From the DFA on the left, we find that $q1$ goes to an accept state on b while $q0$ does not. These states are therefore distinguishable. Highlight one child node and click on $q1$ on the left. Do the same for the other node and click on $q0$. This puts 1 in the left node and 0 in the right.



5. Experiment with other nodes similarly. The correct current partitions are as shown below.



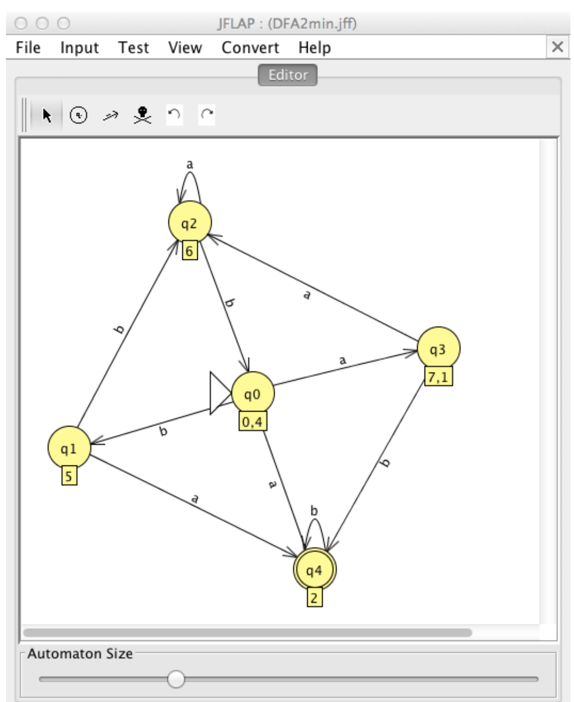
6. We continue this process at the next partition $\{0, 4, 5, 6\}$. Experiment with this. Your answer should be as below.



7. At this stage, JFLAP offers no further expansions. Click on Finish and experiment with adding transitions to the 5-state DFA that is presented to you. Use the buttons Hint and Done? for help.

8. Your final answer should be as below.

9. Test this minimum DFA and the one you stated with to verify that they accept/reject in the same way. Use multiple inputs.



4 References

1. Introduction to the Theory of Computation (Third Edition), Michael Sipser. Cengage Learning. 2013.

2. JFLAP - An Interactive Formal Languages and Automata Package, Susan H. Rodger and Thomas W Finley. Jones and Bartlett Publishers. 2006

3. Automata Theory, Languages, and Computation (3rd Ed), Hopcroft, Motwani and Ullman. Addison Wesley. 2006.