Regular Pumping Lemma

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1 Regular Pumping Lemma

Regular pumping lemma is a powerful tool that helps you show that certain languages are not regular. The statement of the regular pumping lemma is as follows:

If L is a regular language, then there exists an integer m such that given a word w in L, with $|w| \geq m$, w can be decomposed as $w = xyz$, with the following properties:

1. $|xy| \leq m$
2. $|y| > 0$, and
3. for each $i \geq 0$, $xy^iz \in L$.

To use the pumping lemma to show that a given language L is not regular, we proceed by assuming that L is regular so that L satisfies the pumping lemma. Then we select an appropriate w in L and show that w cannot be decomposed as prescribed by the pumping lemma. This contradiction shows that our assumption must be wrong and hence L is not regular.

We will use the language $L = \{w \in \{a,b\}^* : n_a(w) < n_b(w)\}$.

Thus L is the language of those words in $\{a,b\}^*$ that have fewer as than bs.

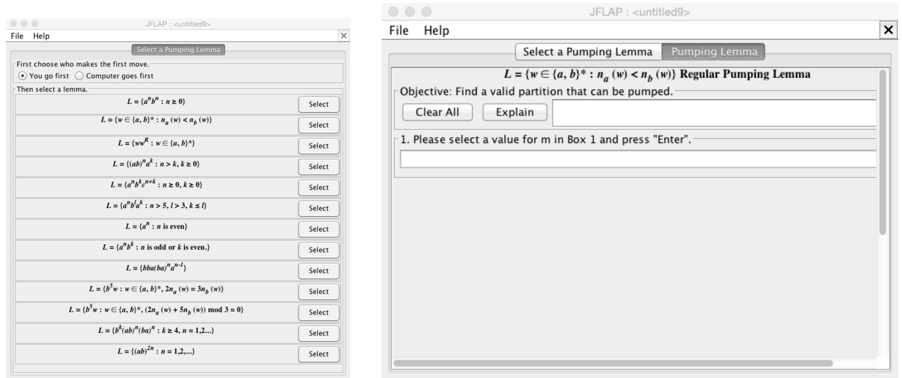
We first give a direct argument to show that L is not regular. Then we use JFLAP to practice showing the same result.

2 Direct Proof

Assume that L is regular. Then L satisfies the pumping lemma and hence there is an integer m such that the three properties above are satisfied. Select $w = a^m b^{m+1}$. Then $w \in L$ and $|w| \geq m$. By pumping lemma, $a^m b^{m+1} = w = xyz$. By property 1. of the pumping lemma, y must be a string of as. Suppose $y = a^k$. By property 2. of the pumping lemma, $k > 0$. Then the string $xy^2z = xyyz = a^{m+k}b^{m+1}$ is not in L. This is a contradiction to property 3. Hence L must be nonregular.

3 Proof with JFLAP

JFLAP provides a proof by letting you play a game against the computer. The idea of the game is to win by providing a word w for which pumping lemma fails. Run JFLAP, select



Language Selection First Step of Proof

Figure 1: Step 1

Regular Pumping Lemma, select "You go first" and then select the language L as above from the list provided. See Figure 1. Click on "Explain" button to see a description of why L is nonregular. Since you go first, JFLAP asks you to enter a number m. Enter 4. See Figure 2.

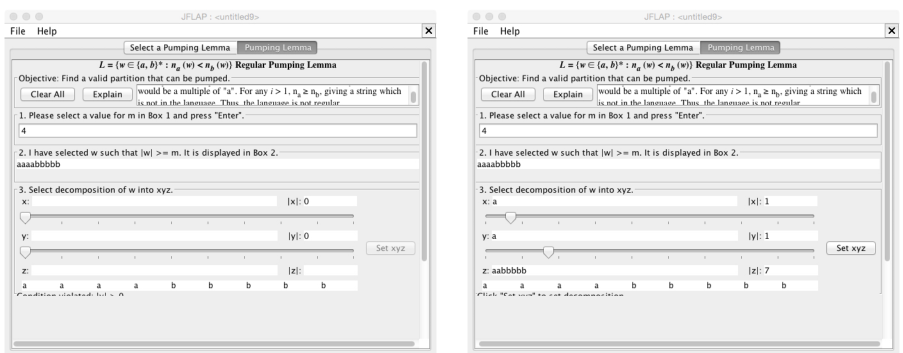


Figure 2: Step 2

At this point JFLAP gives you the word $w = a^4b^5$ and asks you to provide a decomposition of w. Use the sliders to select x and y. Because of properties 1. and 2. of the pumping lemma, y must be one of $a, aa, aaa, aaaa$. Select $y = aa$ and then select $x = a$. z is then selected for you. If x and y are selected appropriately, then the button "Select xyz" can be clicked. Click it. JFLAP then presents you with $i = 2$ and shows that the string $xyyz$ is not in L. You can animate that proof. See Figure 3. Repeat this entire process for some other values of m.

We now do this again but this time we'll let the "Computer go first." Suppose it selects 15 as in Figure 4. You are asked to select a w. Make sure that w is in L and $|w| \geq 15$. What is an appropriate w? See Figure 4. With the right choice of w and then an i, you win. Try choosing different ws. Repeat the entire process two or three times.

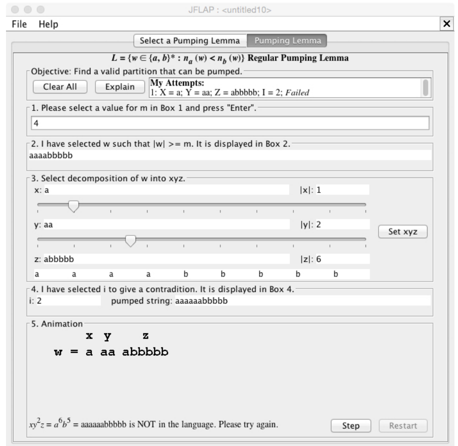


Figure 3: Step 3

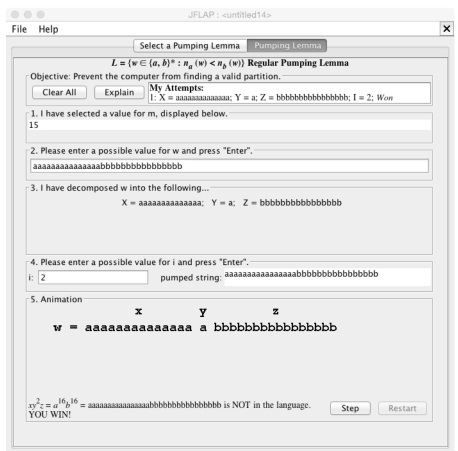


Figure 4: Computer goes first

4 References

1. Introduction to the Theory of Computation (Third Edition), Michael Sipser. Cengage Learning. 2013.

2. JFLAP - An Interactive Formal Languages and Automata Package, Susan H. Rodger and Thomas W Finley. Jones and Bartlett Publishers. 2006